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A BARRELLED SPACE WITHOUT A BASIS

N. J. KALTON

ABSTRACT. An example is given of a separable, barrelled, nuclear, bornological Ptak space which has no Schauder basis.

A sequence $\{x_n\}$ in a locally convex space E is a basis if each x in E can be expressed uniquely in the form $x = \sum_{i=1}^{\infty} a_i x_i$; Singer [5] has given an example of a separable locally convex space which does not possess a basis. In this note I shall give another example which has the additional properties of being barrelled, bornological, nuclear and a Ptak space; this also partially answers a question raised in [2].

It seems desirable to introduce another form of separability in locally convex spaces: E will be called ω -separable if it possesses a subspace G of countable dimension and such that every member of E is the limit of a sequence in G . Thus if E has a basis, E is ω -separable, and if E is ω -separable then E is separable. Let \aleph denote the cardinal of the continuum and let $\text{card}(X)$ denote the cardinal of any set X . If E is ω -separable then $\text{card}(E)$ is less than the cardinal of the set of all sequences in G ; as $\text{card}(G) = \aleph$, $\text{card}(E) = \aleph$.

Let K be the field of real numbers or of complex numbers.

THEOREM. K^{\aleph} is barrelled, bornological, nuclear and a Ptak space; it is separable but not ω -separable, and so does not possess a basis.

PROOF. K^{\aleph} has a weak topology and is complete; hence K^{\aleph} is a Ptak space (see [4, p. 162]). By the Mackey-Ulam theorem ([3, §28.8]) it is bornological as \aleph is not strongly inaccessible; a complete bornological space is barrelled. The product of nuclear spaces is nuclear [4, p. 102] and so K^{\aleph} is nuclear. Finally, K^{\aleph} is separable by Theorem 7.2, p. 175 of [1] but $\text{card}(K^{\aleph}) = 2^{\aleph} > \aleph$, and so K^{\aleph} cannot be ω -separable.

The question which naturally arises is: does every ω -separable locally convex space possess a basis? Singer's example, the weak*-dual of the Banach space m of all bounded sequences, is not ω -separable;¹ this follows from the results of [5] or from the fact that m^* contains a copy of the Stone-Čech compactification of the integers, and so has cardinality 2^{\aleph} .

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