Workshop in Analysis and Probability
Concentration Week

Non-Linear Geometry of
Banach Spaces, Geometric Group
Theory, and Differentiability

August 1-5, 2011

Abstracts

Fernando Albiac

The role of local convexity in Lipschitz maps

With an eye on the nonlinear classification of quasi-Banach spaces, in this talk we gather a few remarks about the role of local convexity in Lipschitz maps. The moral will be that when we naturally attempt to translate properties of Lipschitz functions that hold in Banach spaces to quasi-Banach spaces, generally such properties imply local convexity, whence they cannot be used in the specific setting of nonlocally convex spaces. We also show two instances where the Lipschitz structure of a quasi-Banach space exhibits some degree of consistency with its linear structure.

Michael Doré

Uniformly purely unrectifiable sets and differentiability

An important notion in geometric measure theory is that of a purely unrectifiable set, that is a set whose intersection with every Lipschitz curve has a 1-d Hausdorff measure of 0. A longstanding open question is whether this notion is equivalent to a stronger-looking property known as uniform pure unrectifiability. We discuss recent insight into this question, obtained from the study of the differentiability properties of Lipschitz functions.

Erik Guentner

Decomposing linear groups

I intend to review the motivation for, and the basic definitions and properties of finite decomposition complexity, a coarse geometric property introduced to study isomorphism conjectures in C*-algebra K-theory and topological rigidity. I intend to explain in some detail why every linear group has finite decomposition complexity.

The talk is based on joint work with Romain Tessera and Guoliang Yu.

Ana Khukhro

Box spaces of groups and uniform embeddings into Hilbert space

Abstract: Given a nitely generated group, we can deduce things about its structure from the geometric properties of its Cayley graph, and vice versa. A box space is another geometric tool which can be associated to a nitely generated residually nite group. It is a metric space made from nite quotients of the group which in some sense approximate the group's Cayley graph. There are many intriguing connections between geometric properties of the box space and analytic properties of the group. We will describe the properties involved, investigate how uniform embeddability of box spaces into Hilbert space behaves under certain group constructions, and look at some open problems.

Denka Kutzarova

On property (β) of Rolewicz and related geometric properties

Recently V. Lima and L. Randrianarivony solved a problem posed in 1999 by Bates, Johnson, Lindenstrauss, Preiss and Schechtman and they made use of the property (β) of S. Rolewicz. As a background for this result, we shall review old results on isometric and isomorphic characterizations of property (β) and the connection to other similar but isomorphically different geometric properties.

Dennis Dreesen

The behaviour of (equivariant) Hilbert space compression under group constructions

Let H be a finitely generated group equipped with the word length metric relative to a finite symmetric generating subset. Uniform embeddability of H into a Hilbert space is an interesting notion since it implies e.g. that H satisfies the coarse Baum-Connes Conjecture [2]. The Hilbert space compression of a group indicates how well a certain group embeds uniformly into a Hilbert space. Here, there are connections with Yu's property (A) [1]. More precisely, the Hilbert space compression of a finitely generated group G is a number between 0 and 1 that describes how close a uniform embedding $f : G \to \ell_2(\mathbb{Z})$ can be to being quasi-isometric. If this number is strictly greater than $\frac{1}{2}$, then the group satisfies Yu's property (A) [1]. The equivariant Hilbert space compression only takes into account those uniform embeddings which are G-equivariant relative to some afine isometric action of G on $\ell_2(\mathbb{Z})$ and the left multiplication action of G on itself. If this number is strictly greater than $\frac{1}{2}$, then the group is amenable [1]. We elaborate on the behaviour of the (equivariant) Hilbert space compression under group constructions such as free products, certain group extensions (e.g. by groups of polynomial growth or hyperbolic groups), and so forth.

References

1. E. Guentner, J. Kaminker, 'Exactness and uniform embeddability of discrete groups', Journal of the London Mathematical Society 70, no. 3 (2004), 703-718.

2. G. Skandalis, J. L. Tu, and G. Yu, 'Coarse Baum-Connes conjecture and groupoids', Topology 41 (2002), 807-834.

Gilles Godefroy

1. Free spaces

The natural predual of the space of Lipschitz functions on a metric space M. The lifting property for separable Banach spaces. Free spaces and the approximation properties. Preduals of spaces of Hölder functions. Canonical pairs of separable Banach spaces which are uniformly homeomorphic but not linearly isomorphic.

2. Uniform homeomorphisms

Asymptotic invariants for Lipschitz and uniform homeomorphisms. Countable Kalton graphs. Some spaces which are determined by their uniform structure. Spaces in which c_0 coarsely embeds and Aharoni's problem.

3. The non-separable theory

The Aharoni–Lindenstrauss example and Lipschitz liftings of non-separable c_0 spaces. WCG spaces and Lipschitz isomorphisms. Uncountable Kalton graphs. There is no uniformly continuous selection of the quotient map from ℓ_∞ onto ℓ_∞/c_0. Banach spaces which are not Lipschitz retracts of their biduals.

Mikhail Ostrovskii

Embeddability of locally finite metric spaces into Banach spaces is finitely determined

The main results presented in the talk are: Let A be a locally finite metric space whose finite subsets admit uniformly bi-Lipschitz embeddings into a Banach space X. Then A admits a bi-Lipschitz embedding into X.

Let A be a locally finite metric space whose finite subsets admit uniformly coarse embeddings into a Banach space X. Then A admits a coarse embedding into X.

These results generalize previously known results of the same type due to Brown–Guentner (2005), Baudier (2007), Baudier-Lancien (2008), and the speaker (2006, 2009).

One of the main steps in the proof is: each locally finite subset of an ultraproduct X^U admits a bi-Lipschitz embedding into X. The speaker is going to present details for the most important step of the proof.

Lova Randrianarivony

Application of property (β) to uniform quotient maps

Rolewicz introduced the property (β) of Banach spaces, for which Kutzarova later gave a characterization. We exploit the metrical language of Kutzarova's characterization to study uniform quotient mappings between ℓ_p-spaces.

This is joint work with Vegard Lima.

Mark Sapir

Coarse embeddings into Banach spaces from the group theory point of view

I will talk about

- Possible Hilbert space and Banach space compression functions of groups including the R. Thompson group F.
- Embeddability of contractible manifolds into Hilbert spaces.

Gideon Schechtman

Tight embedding of subspaces of L_p in ℓ_p^n for even p

Given $1 \leq p < \infty$ and k what is the minimal n such that ℓ_p^n almost isometrically contain all k-dimensional subspaces of L_p? I'll survey what is known about this problem and then concentrate on a recent result, basically solving the problem for even p. The proof uses a recent result of Batson, Spielman and Srivastava.

Gilles Lancien

A few remarks around Godefroy–Kalton's theorem on isometric embeddings

As an application of their fundamental work on Lipschitz free spaces, that appeared in Studia Mathematica in 2003, G. Godefroy and N.J. Kalton obtained the following striking result. If a separable Banach space X isometrically embeds into a Banach space Y, then it embeds linearly and isometrically into Y. The first ingredient of the proof is a result of Figiel which insures that if j is an isometry from X into Y (with $j(0) = 0$), then there exists a linear quotient map Q of norm 1 from the closed linear span of $j(X)$ onto X such that $Q \circ j = I_X$. Then the work of Godefroy and Kalton implies that in this situation Q has a linear right inverse of norm 1, which yields the conclusion.

In this talk we will consider the following related question. Assume that X is a separable Banach space and that the unit ball of X (or more generally a "small" but total subset of X) embeds isometrically into a Banach space Y. Does it imply that X embeds linearly isometrically into Y? We will give partial answers to this question. In particular, we will give examples and counterexamples for the validity of Figiel's theorem in this setting.

James Lee

Discrete differentiation and local rigidity of smooth sets in the plane

We exhibit an infinite doubling space whose has n-point subsets requiring bi-Lipschitz distortion $\sim (log\ n)^{1/2}$ to embed into L_1, matching the upper bound of [Gupta-Krauthgamer-Lee 2003]. This improves over the best previous bound of $(log\ n)^c$ for some $c > 0$ [Cheeger–Kleiner–Naor 2009]. Furthermore, this offers a nearly tight integrality gap for a weak version of the Goemans-Linial SDP for Sparsest Cut, matching an upper bound of [Arora-Lee-Naor 2005].

Following the general approach developed by Cheeger and Kleiner (2006), our lower bound uses a differentiation argument to achieve local control on the cut measure, followed by a classification of the cuts that can appear. The main technical difficulty involves approximately classifying certain kinds of weakly regular sets in the plane. This weak regularity is achieved via a random differentiation argument which measures the variation of the function along randomly chosen subdivisions of geodesics.

Our lower bound space is a 3-dimensional cube complex which takes inspiration from both the Heisenberg group and the diamond graphs.

This is joint work with Tasos Sidiropoulos (Toyota Institute of Technology).

Alain Valette

Euclidean distortion and spectral gap for finite, regular graphs

For finite, connected, regular graphs X, we prove that the euclidean distortion is bounded below by the square root of the normalized spectral gap, times the displacement of X (that is, the maximal displacement of permutations of vertices of X). This inequality allows us to give unified proofs for computations of euclidean distortion for cubes (Enflo 1969), cycles (Linial–Magen 2001), and k-regular expanders (Linial–London–Rabinovich 1995). It also allows for non-trivial lower bounds on distortion for some non-expanding families of regular graphs, e.g. Cayley graphs of $SL_n(p)$ (p fixed, n varying). This is joint work with Pierre-Nicolas Jolissaint.

Rufus Willett

Girth of graphs, property (T) and higher index theory

I will describe joint work with Guoliang Yu. We study sequences of graphs with girth tending to infinity – these are of interest, for example, as they are necessary for the construction of the 'Gromov monster' groups which have bad coarse embedding properties. From the point of view of higher index theory, we show that such a family is in some sense relatively tractable. On the other hand, we introduce a notion of property (T) for a sequence of graphs, which is strictly stronger than the family being an expander, and an obstruction to such results.