

# A remark on the $H^\infty$ -calculus

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## Abstract

If  $A, B$  are sectorial operators on a Hilbert space with the same domain and range, and if  $\|Ax\| \approx \|Bx\|$  and  $\|A^{-1}x\| \approx \|B^{-1}x\|$ , then it is a result of Auscher, McIntosh and Nahmod that if  $A$  has an  $H^\infty$ -calculus then so does  $B$ . On an arbitrary Banach space this is true with the additional hypothesis on  $B$  that it is almost R-sectorial as was shown by the author, Kunstmann and Weis in a recent preprint. We give an alternative approach to this result.

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## 1 Introduction

In [1] the authors showed that if  $X$  is a Hilbert space and  $A, B$  are sectorial operators with the same domain and range and satisfying estimates

$$\|Ax\| \approx \|Bx\| \quad x \in \text{Dom}(A) \quad (1.1)$$

and

$$\|A^{-1}x\| \approx \|B^{-1}x\| \quad x \in \text{Ran}(A) \quad (1.2)$$

then if one of  $(A, B)$  admits an  $H^\infty$ -calculus then so does the other. Results of this type are useful in applications and were studied in [7] for arbitrary Banach spaces. In that paper, a similar result (Theorem 5.1) is proved under the additional hypothesis that  $A$  is almost R-sectorial.

In this note we give a rather different approach to this result. We replace the almost R-sectoriality assumption by the technically weaker assumption of almost U-sectoriality, although this is probably not of great significance. However, our approach here is perhaps a little simpler. We also point out

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that some additional assumption is necessary in arbitrary Banach spaces; there are examples of sectorial operators  $A, B$  satisfying (1.1) and (1.2) but such that only one has an  $H^\infty$ -calculus.

It is possible to consider estimates on fractional powers and our results can be extended in this direction (as in [7]); however to keep the exposition simple we will not discuss this point. We also point out that our approach is really based on an interpolation method, known as the Gustavsson-Peetre method [5] (see also [4]); but to avoid certain technicalities we have not made this explicit.

## 2 U-bounded collections of operators

Let  $X$  be a complex Banach space. A family  $\mathcal{T}$  of operators  $T : X \rightarrow X$  is called *U-bounded* if there is a constant  $C$  such that if  $(x_j)_{j=1}^n \subset X$ ,  $(x_j^*)_{j=1}^n \subset X^*$ ,  $(T_j)_{j=1}^n \subset \mathcal{T}$ ,

$$\sum_{j=1}^n |\langle T_j x_j, x_j^* \rangle| \leq C \sup_{|a_j|=1} \left\| \sum_{j=1}^n a_j x_j \right\| \sup_{|a_j|=1} \left\| \sum_{j=1}^n a_j x_j^* \right\|.$$

The best such constant  $C$  is called the U-bound for  $\mathcal{T}$  and is denoted  $U(\mathcal{T})$ . This concept was introduced in [8].

We recall that  $\mathcal{T}$  is called *R-bounded* if there is a constant  $C$  such that if  $(x_j)_{j=1}^n \subset X$ ,  $(T_j)_{j=1}^n \subset \mathcal{T}$ ,

$$(\mathbb{E} \left\| \sum_{j=1}^n \epsilon_j T x_j \right\|^2)^{1/2} \leq C (\mathbb{E} \left\| \sum_{j=1}^n \epsilon_j x_j \right\|^2)^{1/2}.$$

Here  $(\epsilon_j)_{j=1}^n$  is a sequence of independent Rademachers. The best such constant  $C$  is called the R-bound for  $\mathcal{T}$  and is denoted  $R(\mathcal{T})$ . An R-bounded family is automatically U-bounded [8].

We will need the following elementary property:

**Proposition 2.1.** *Suppose  $F : (0, \infty) \rightarrow \mathcal{L}(X)$  is a continuous function and that  $\mathcal{T} = \{F(t) : 0 < t < \infty\}$  is U-bounded with U-bound  $U(F)$ . Suppose  $g \in L_1(\mathbb{R}, dt/t)$ . Then the family of operators*

$$G(s) = \int_0^\infty g(st) F(t) \frac{dt}{t} \quad 0 < s < \infty$$

*is U-bounded with constant at most  $U(F) \int_0^\infty |g(t)| dt/t$ .*

*Proof.* Suppose  $(x_j)_{j=1}^n \subset X$ ,  $(x_j^*)_{j=1}^n \subset X^*$  with

$$\sup_{|a_j|=1} \left\| \sum_{j=1}^n a_j x_j \right\|, \sup_{|a_j|=1} \left\| \sum_{j=1}^n a_j x_j^* \right\| \leq 1.$$

Then for  $s_1, \dots, s_n \in \mathbb{R}$  we have

$$\begin{aligned} \sum_{j=1}^n |\langle G(s_j) x_j, x_j^* \rangle| &\leq \sum_{j=1}^n \int_0^\infty |g(t)| |\langle F(s_j^{-1} t) x_j, x_j^* \rangle| \frac{dt}{t} \\ &\leq U(F) \int_0^\infty |g(t)| \frac{dt}{t}. \end{aligned}$$

□

### 3 Sectorial operators

Let  $X$  be a complex Banach space and let  $A$  be a closed operator on  $X$ .  $A$  is called *sectorial* if  $A$  has dense domain  $\text{Dom}(A)$  and dense range  $\text{Ran}(A) = \text{Dom}(A^{-1})$  and for some  $0 < \varphi < \pi$  the resolvent  $(\lambda - A)^{-1}$  is bounded for  $|\arg \lambda| \geq \varphi$  and satisfies the estimate

$$\sup_{|\arg \lambda| \geq \varphi} \|\lambda(\lambda - A)^{-1}\| < \infty.$$

The infimum of such angles  $\varphi$  is denoted  $\omega(A)$ .

Let  $\Sigma_\varphi$  be the open sector  $\{z \neq 0 : |\arg z| < \varphi\}$ . If  $f \in H^\infty(\Sigma_\varphi)$  we say that  $f \in H_0^\infty(\Sigma_\varphi)$  if there exists  $\delta > 0$  such that  $|f(z)| \leq C \max(|z|^\delta, |z|^{-\delta})$ . For  $f \in H_0^\infty(\Sigma_\varphi)$  where  $\varphi > \omega(A)$  we can define  $f(A)$  by a contour integral, which converges as a Bochner integral in  $\mathcal{L}(X)$ .

$$f(A) = \frac{1}{2\pi i} \int_{\Gamma_\nu} f(\lambda)(\lambda - A)^{-1} d\lambda$$

where  $\Gamma_\nu$  is the contour  $\{|t|e^{-i\nu \text{sgn } t} : -\infty < 0 < \infty\}$  and  $\omega(A) < \nu < \varphi$ . We can then estimate  $\|f(A)\|$  by

$$\|f(A)\| \leq C_\varphi \int_{\Gamma_\nu} |f(\lambda)| \frac{|d\lambda|}{|\lambda|}.$$

If we have a stronger estimate

$$\|f(A)\| \leq C\|f\|_{H^\infty(\Sigma_\varphi)} \quad f \in H_0^\infty(\Sigma_\varphi)$$

then we say that  $A$  has an  $H^\infty(\Sigma_\varphi)$ -calculus; in this case we may extend the functional calculus to define  $f(A)$  for every  $f \in H^\infty(\Sigma_\varphi)$ . The infimum of all such angles  $\varphi$  is denoted by  $\omega_H(A)$ .

We will need a criterion for the existence of an  $H^\infty$ -calculus. It will be convenient to use the notation  $f_\lambda(z) = f(\lambda z)$  and to let  $u(z) = z(1+z)^{-2}$  so that  $u \in H_0^\infty(\Sigma_\varphi)$  for all  $\varphi < \pi$ . The following criterion goes back to [2] and [3]. A simple proof is given in [10].

**Proposition 3.1.** *Let  $A$  be a sectorial operator and suppose  $0 < \varphi < \pi$ . Then the following are equivalent:*

(i) *There is a constant  $C$  so that*

$$\int_0^\infty |\langle u_\mu(tA)x, x^* \rangle| \frac{dt}{t} \leq C\|x\|\|x^*\| \quad |\arg \mu| = \varphi, \quad x \in X, x^* \in X^*.$$

(ii)  *$A$  has an  $H^\infty$ -calculus with  $\omega_H(A) \leq \pi - \varphi$ .*

*Remark.* (i) is equivalent by the Maximum Modulus Principle to

$$\int_0^\infty |\langle u_\mu(tA)x, x^* \rangle| \frac{dt}{t} \leq C\|x\|\|x^*\| \quad |\arg \mu| \leq \varphi, \quad x \in X, x^* \in X^*.$$

If  $A$  is sectorial we can define a closed operator  $A^*$  on  $X^*$  by  $A^*x^* = x^* \circ A$  with domain  $\text{Dom}(A^*)$  consisting of all  $x^*$  such that  $x \rightarrow x^*(Ax)$  extends to a bounded linear functional on  $X$ . Then  $A^*$  need not be sectorial since it need not have dense domain or range. Note that

$$\|A^*x^*\| = \sup_{\substack{\|A^{-1}x\| \leq 1 \\ x \in \text{Ran}(A)}} |\langle x, x^* \rangle| \quad x^* \in \text{Dom}(A^*)$$

and

$$\|(A^*)^{-1}x\| = \sup_{\substack{\|Ax\| \leq 1 \\ x \in \text{Dom}(A)}} |\langle x, x^* \rangle| \quad x^* \in \text{Ran}(A^*).$$

Thus if  $A$  and  $B$  are sectorial operators satisfying (1.1) and (1.2) they will also satisfy  $\text{Dom}(A^*) = \text{Dom}(B^*)$ ,  $\text{Ran}(A^*) = \text{Ran}(B^*)$  and

$$\|A^*x^*\| \approx \|B^*x^*\| \quad x^* \in \text{Dom}(A^*) \quad (3.1)$$

and

$$\|(A^*)^{-1}x^*\| \approx \|(B^*)^{-1}x^*\| \quad x^* \in \text{Ran}(A^*) \quad (3.2)$$

If  $A$  is a sectorial operator and  $\varphi > \omega(A)$  we shall that  $f \in H_0^\infty(\Sigma_\varphi)$  is U-bounded (respectively R-bounded) for  $A$  if the family of operators  $\{f(tA) : 0 < t < \infty\}$  is a U-bounded (respectively R-bounded) collection.

**Proposition 3.2.** *Suppose  $A$  has an  $H^\infty$ -calculus and that  $\varphi > \omega_H(A)$ . Then for any  $f \in H_0^\infty(\Sigma_\varphi)$  we have that  $f$  is R-bounded (and thus U-bounded) for  $A$ .*

*Proof.* Suppose  $\omega(A) < \psi < \varphi$ . Then the map  $\lambda \rightarrow f(\lambda A)$  is analytic on  $\Sigma_{\varphi-\psi}$  and extends continuously to the boundary. The operators  $\{f(2^k t e^{\pm i(\varphi-\psi)} A)\}_{k \in \mathbb{Z}}$  are R-bounded (uniformly in  $0 < t < \infty$ ) by Theorem 3.3 of [8] and the result follows by Lemma 3.4 of the same paper.  $\square$

Suppose  $A$  is a sectorial operator on  $X$  and  $\varphi > \omega(A)$ . We will say that  $A$  is *almost U-sectorial* (respectively *almost R-sectorial*) if there is an angle  $\varphi$  such that the set of operators  $\{\lambda A R(\lambda, A)^2 : |\arg \lambda| \geq \varphi\}$  is U-bounded (respectively R-bounded). If we define  $u(z) = z(1+z)^{-2}$  this implies that the functions  $u_\lambda(z) = u(\lambda z)$  are uniformly U-bounded (respectively uniformly R-bounded) for  $|\arg \lambda| \leq \pi - \varphi$ . The infimum of such angles is denoted  $\tilde{\omega}_U(A)$ . By Lemma 3.4 of [8] this definition is equivalent to

$$\tilde{\omega}_U(A) = \pi - \sup\{\theta : u_{e^{\pm i\theta}} \text{ is U-bounded}\}$$

or, respectively

$$\tilde{\omega}_R(A) = \pi - \sup\{\theta : u_{e^{\pm i\theta}} \text{ is R-bounded}\}.$$

**Proposition 3.3.** *Suppose  $A$  admits an  $H^\infty$ -calculus. Then  $A$  is almost R-sectorial (and hence almost U-sectorial) and  $\tilde{\omega}_U(A) \leq \tilde{\omega}_R(A) \leq \omega_H(A)$ .*

*Proof.* This follows from Proposition 3.2.  $\square$

**Lemma 3.1.** *Suppose  $A$  is almost U-sectorial and  $\varphi > \nu > \tilde{\omega}_U(A)$ . Then there is a constant  $C = C(\varphi)$  so that if  $f \in H_0^\infty(\Sigma_\varphi)$  then  $f$  is U-bounded for  $A$  with U-bound*

$$U(f) \leq C \int_{\Gamma_\nu} |f(\lambda)| \frac{|d\lambda|}{|\lambda|}.$$

*Proof.* Fix  $\varphi > \psi > \nu > \omega_U(A)$ . We may write  $f(tA)$  in the form

$$f(tA) = \frac{1}{2\pi i} \int_{\Gamma_\psi} f(t\lambda) \lambda^{-1/2} A^{1/2} (\lambda - A)^{-1} d\lambda.$$

Therefore the result follows from Lemma 2.1 once we show that the two families of operators  $\{h(e^{\pm i\theta} tA) : 0 < t < \infty\}$  are U-bounded where  $\theta = \pi - \psi$  and  $h(z) = z^{1/2}(1+z)^{-1}$ .

Consider

$$g(z) = -i \log \frac{1 + iz^{1/2}}{1 - iz^{1/2}} - \pi \frac{z}{1+z} \quad |\arg z| < \pi.$$

Then  $g \in H_0^\infty(\Sigma_\pi)$ . Furthermore

$$g'(z) = z^{-1/2}(1+z)^{-1} - \pi(1+z)^{-2}.$$

Hence  $g_{e^{\pm i\theta}} \in H_0^\infty(\Sigma_\psi)$ . For convenience we consider the case of  $+\theta$ . Thus if

$$T_t = -\frac{1}{2\pi i} \int_{\Gamma_\nu} g(te^{i\theta} \lambda) A (\lambda - A)^{-2} d\lambda$$

the family of operators  $\{T_t : 0 < t < \infty\}$  is U-bounded, again by Lemma 2.1. Now integration by parts shows that

$$\begin{aligned} T_t &= \frac{te^{i\theta}}{2\pi i} \int_{\Gamma_\nu} ((te^{i\theta} \lambda)^{-1/2} (1 + te^{i\theta} \lambda)^{-1} - \pi(1 + te^{i\theta} \lambda)^{-2}) \lambda (\lambda - A)^{-1} d\lambda \\ &= \frac{1}{2\pi i} \int_{\Gamma_\nu} (h(te^{i\theta} \lambda) - \pi u(te^{i\theta} \lambda)) (\lambda - A)^{-1} d\lambda \\ &= h(te^{i\theta} A) - \pi u(te^{i\theta} A). \end{aligned}$$

Thus it follows that the family  $\{h(te^{i\theta} A) : 0 < t < \infty\}$  is U-bounded.  $\square$

## 4 The main results

If  $A$  is sectorial then the space  $\text{Dom}(A) \cap \text{Ran}(A)$  is a Banach space (densely) embedded into  $X$  under the norm  $\|Ax\| + \|A^{-1}x\| + \|x\|$ ; similarly  $\text{Dom}(A^*) \cap \text{Ran}(A^*)$  is a Banach space embedded into  $X^*$  under the norm  $\|A^*x^*\| + \|(A^*)^{-1}x^*\| + \|x^*\|$ .

**Theorem 4.1.** *Suppose  $A$  is a sectorial operator. In order that  $A$  have an  $H^\infty$ -calculus with  $\omega_H(A) = \varphi$  it is necessary and sufficient that:*

- (i)  $A$  is almost  $U$ -sectorial with  $\tilde{\omega}_U(A) = \varphi$ .
- (ii) There exists a constant  $C_1$  so that for each  $x \in X$  there is a continuous function  $\xi : (0, \infty) \rightarrow \text{Dom}(A) \cap \text{Ran}(A)$  such that

$$\left\| \sum_{k=-N}^N a_k 2^{jk} t^j A^j \xi(2^k t) \right\| \leq C_1 \|x\|, \quad j = -1, 0, 1, \quad |a_k| \leq 1, \quad N = 1, 2, \dots, \quad 0 < t < \infty$$

and

$$\langle x, x^* \rangle = \int_0^\infty \langle \xi(t), x^* \rangle \frac{dt}{t} \quad x^* \in X^*.$$

- (iii) There exists a constant  $C_2$  so that for each  $x^* \in X^*$  there is a continuous function  $\xi^* : (0, \infty) \rightarrow \text{Dom}(A^*) \cap \text{Ran}(A^*)$  such that

$$\left\| \sum_{k=-N}^N a_k 2^{jk} t^j (A^j)^* \xi^*(2^k t) \right\| \leq C_2 \|x^*\|, \quad j = -1, 0, 1, \quad |a_k| \leq 1, \quad N = 1, 2, \dots, \quad 0 < t < \infty$$

and

$$\langle x, x^* \rangle = \int_0^\infty \langle x, \xi^*(t) \rangle \frac{dt}{t} \quad x \in X.$$

*Proof.* Let us assume (i), (ii) and (iii). Suppose  $|\theta| < \pi - \varphi$  and  $\|x\| \leq 1$ ,  $\|x^*\| \leq 1$ . Let  $\xi(t), \xi^*(t)$  be chosen according to (ii) and (iii). We define  $\tilde{\xi}(t) = tA\xi(t) + t^{-1}A^{-1}\xi(t) + 2\xi(t)$ ,  $\tilde{\xi}^*(t) = tA^*\xi^*(t) + t^{-1}A^*\xi^*(t) + 2\xi^*(t)$ .

Thus we have

$$\left\| \sum_{k=-N}^N a_k 2^{jk} \tilde{\xi}(2^k t) \right\| \leq 3C_1, \quad j = -1, 0, 1, \quad |a_k| \leq 1, \quad N = 1, 2, \dots, \quad 0 < t < \infty$$

and

$$\left\| \sum_{k=-N}^N a_k 2^{jk} \tilde{\xi}^*(2^k t) \right\| \leq 3C_2, \quad j = -1, 0, 1, \quad |a_k| \leq 1, \quad N = 1, 2, \dots, \quad 0 < t < \infty.$$

Note that  $\tilde{\xi} : (0, \infty) \rightarrow X$  and  $\tilde{\xi}^* : (0, \infty) \rightarrow X^*$  are both continuous and

$$\begin{aligned} \xi(t) &= u(tA)\tilde{\xi}(t) & 0 < t < \infty \\ \xi^*(t) &= (u(tA))^*\tilde{\xi}^*(t) & 0 < t < \infty. \end{aligned}$$

If  $\pi - |\arg \mu| > \nu > \varphi$  we have

$$\begin{aligned} \int_0^\infty |\langle u_\mu(rA)x, x^* \rangle| \frac{dr}{r} &\leq \int_0^\infty \int_0^\infty \int_0^\infty |\langle u_\mu(rA)\xi(s), \xi^*(t) \rangle| \frac{dt}{t} \frac{ds}{s} \frac{dr}{r} \\ &= \int_0^\infty \int_0^\infty \int_0^\infty |\langle u_\mu(rtA)\xi(st), \xi^*(t) \rangle| \frac{dt}{t} \frac{ds}{s} \frac{dr}{r} \end{aligned}$$

For fixed  $r, s$

$$\begin{aligned} \int_0^\infty |\langle u_\mu(rtA)\xi(st), \xi^*(t) \rangle| \frac{dt}{t} &= \int_0^\infty |\langle u_\mu(rtA)u(stA)\tilde{\xi}(st), (u(tA))^* \tilde{\xi}^*(t) \rangle| \frac{dt}{t} \\ &= \int_1^2 \sum_{j \in \mathbb{Z}} |\langle u_{r\mu}(2^j tA)u_s(2^j tA)u(2^j tA)\tilde{\xi}(s2^j t), \tilde{\xi}^*(2^j t) \rangle| \frac{dt}{t} \\ &\leq 9C_1 C_2 U(u_{r\mu} u_s u) \\ &\leq C \int_{\Gamma_\nu} |u(r\mu\lambda)u(s\lambda)u(\lambda)| \frac{|d\lambda|}{|\lambda|}, \end{aligned}$$

where  $C$  is constant independent of  $x, x^*$ . Integrating over  $r, s$  gives:

$$\int_0^\infty |\langle u_\mu(rA)x, x^* \rangle| \frac{dr}{r} \leq C \left( \int_{\Gamma_\nu} |u_\mu(\lambda)| \frac{|d\lambda|}{|\lambda|} \right) \left( \int_{\Gamma_\nu} |u(\lambda)| \frac{|d\lambda|}{|\lambda|} \right)^2.$$

This estimate shows, by Proposition 3.1, that  $A$  has an  $H^\infty$ -calculus with  $\omega_H(A) \leq \varphi$ . Since  $\tilde{\omega}_U(A) \leq \omega_H(A)$  by Proposition 3.3 we have equality.

To complete the proof we show that if  $A$  has an  $H^\infty$ -calculus then (i), (ii) and (iii) hold and that  $\tilde{\omega}_U(A) \leq \omega_H(A)$ .

To show (ii) and (iii) we observe that

$$12 \int_0^\infty (u(tz))^2 \frac{dt}{t} = 1.$$

Note that  $z^j u(z)^2 \in H_0^\infty(\Sigma_\varphi)$  for  $j = -1, 0, 1$ . It follows easily that if  $x \in X$  and  $x^* \in X^*$  then

$$\xi(t) = 12u(tA)^2 x, \quad \xi^*(t) = 12(u(tA)^2)^* x^*$$

give the required functions.

For (i) observe that  $\tilde{\omega}_U(A) \leq \omega_H(A)$  but the first part of the proof shows equality. □



**Theorem 4.2.** *Suppose  $A$  and  $B$  are sectorial operators such that  $\text{Dom}(A) = \text{Dom}(B)$ ,  $\text{Ran}(A) = \text{Ran}(B)$  and for a suitable constant  $C$  we have*

$$C^{-1}\|Ax\| \leq \|Bx\| \leq C\|Ax\| \quad x \in \text{Dom}(A)$$

and

$$C^{-1}\|A^{-1}x\| \leq \|B^{-1}x\| \leq C\|A^{-1}x\| \quad x \in \text{Ran}(A).$$

*Suppose  $A$  has an  $H^\infty$ -calculus. Then the following are equivalent:*

(i)  *$B$  has an  $H^\infty$ -calculus with  $\omega_H(B) = \varphi$ .*

(ii)  *$B$  is almost  $U$ -sectorial and  $\tilde{\omega}_U(B) = \varphi$ .*

*Proof.* This is now immediate from Theorem 4.1 using (3.1) and (3.2).  $\square$

If  $X$  is a Hilbert space then the assumption that  $B$  is almost  $U$ -sectorial is redundant and this reduces to the result of Auscher, McIntosh and Nahmod [1]. However, in general this assumption cannot be eliminated. It suffices to take a sectorial operator  $A$  with an  $H^\infty$ -calculus with  $\omega_H(A) > \omega(A)$ . Such examples exist [6]; in fact examples are known on subspaces of  $L_p$  when  $1 < p < 2$  [9]. Now fix  $\theta$  with  $\pi - \omega_H(A) < \theta < \pi - \omega(A)$ . Thus  $e^{\pm i\theta}A$  are sectorial with  $\omega(e^{\pm i\theta}A) \leq \omega(A) + \pi - \theta$ . However if both have an  $H^\infty$ -calculus we would deduce that for a suitable constant  $C$

$$\int_0^\infty |\langle u(te^{\pm i\theta}A)x, x^* \rangle| \frac{dt}{t} \leq C\|x\|\|x^*\| \quad x \in X, x^* \in X^*$$

which would imply that  $\omega_H(A) \leq \pi - \theta$ . This contradiction implies that at least one of  $e^{\pm i\theta}A$  fails to have an  $H^\infty$ -calculus. However if  $B = e^{\pm i\theta}A$  then (1.1) and (1.2) are trivially satisfied.

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### Preface

This volume contains the proceedings of the CMA/AMSI Research Symposium on "Asymptotic Geometric Analysis, Harmonic Analysis and Related Topics", organized by Andrew Hassell, Alan McIntosh, Shahar Mendelson, Pierre Portal, and Fyodor Sukochev at Murramarang (NSW) in February 2006. The meeting was sponsored by the Centre for Mathematics and its Applications (Australian National University) and the Australian Mathematical Sciences Institute whose support is gratefully acknowledged.

The Symposium covered a variety of topics in functional, geometric, and harmonic analysis, and brought together experts, early career researchers, and doctoral students from Australia, Canada, Finland, France, Germany, Israel, and the USA. It is our hope that this volume reflects the lively research atmosphere of this conference, and we are glad to open it with a result of Ian Doust, Florence Lancien, and Gilles Lancien, which was essentially discovered during the symposium.

We also wish to express our appreciation to the participants, the authors who contributed to this volume, and the CMA support staff (Chris Wetherell and Annette Hugues) who made this symposium what it was.






Each article in this volume was peer refereed.

Alan McIntosh and Pierre Portal  
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