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A remark on sectorial operators with an H^∞–calculus

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Abstract. We construct examples of sectorial operators admitting an H^∞-calculus so that the angle of sectoriality and the angle of the H^∞–calculus are different.

1. Introduction

Let X be a complex Banach space. A *sectorial* operator A on X is a one-one closed operator with dense domain and range such that the resolvent operator $R(\lambda, A) = (\lambda - A)^{-1}$ is defined and bounded outside a sector $|\arg \lambda| \le \phi$ and further satisfies an estimate

$$\|\lambda R(\lambda, A)\| \le C \qquad |\arg \lambda| \ge \phi. \tag{1.1}$$

The infimum of all ϕ so that (1.1) holds is denoted by $\omega(A)$. Let us recall that a closed operator is of *type* ω if its resolvent is well-defined outside a sector and satisfies an estimate of type (1.1). Such an operator becomes sectorial if in addition we have that $\lim_{t\to 0-} tR(t, A)x = 0$ and $\lim_{t\to -\infty} tR(t, A)x = x$ for every $x \in X$.

If A is sectorial it is possible to define a functional calculus for certain functions bounded and analytic on a sector $\Sigma_\phi = \{\lambda : |\arg \lambda| < \phi\}$ where $\phi > \omega(A)$. We refer to [**2**] for details. We say that A admits an $H^\infty(\Sigma_\phi)$–calculus if $f(A)$ is a bounded operator for every $f \in H^\infty(\Sigma_\phi)$. If A admits an H^∞–calculus for some $0 < \phi < \pi$ we define $\omega_H(A)$ to be the infimum of all such ϕ.

A basic result due to McIntosh [**4**] is that if X is a Hilbert space and A admits an H^∞–calculus for some angle then $\omega_H(A) = \omega(A)$. In [**2**] the question is asked whether this is true in an arbitrary Banach

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space. There is an example in [**2**] (Example 5.5) which almost answers this question negatively; it is, however, not a sectorial operator because it fails to have dense range.

The object of this note is to give a natural counterexample to the question in [**2**]. For $0 < \theta < \pi$ we construct a sectorial operator with $\omega(A) = 0$ and $\omega_H(A) = \theta$. By an interpolation argument we show that we can choose X to be uniformly convex.

Unfortunately we do not know an example on an explicit space such as L_p when $1 < p < \infty$ and $p \neq 2$.

2. The examples

We start with the space $L_2(\mathbb{R})$. It will be convenient to norm this space by

$$\|f\|_0^2 = 2\pi \int_{-\infty}^{\infty} |f(x)|^2 dx = \int_{-\infty}^{\infty} |\hat{f}(\xi)|^2 d\xi$$

where $\hat{f}$ is the Fourier transform. The identity follows by Plancherel's theorem. On this space we define a sectorial operator A by

$$Af(x) = e^x f(x)$$

with domain $\mathcal{D}(A) = \{f : \ e^x f(x) \in L_2\}$. It is clear that A is sectorial with $\omega(A) = 0$. In fact A has an H^∞-calculus and $\omega_H(A) = 0$.

For $\theta > 0$ we define a Euclidean norm on L_2 by

$$\|f\|_\theta^2 = \int_{-\infty}^{\infty} e^{-2\theta|\xi|} |\hat{f}(\xi)|^2 d\xi.$$

Let $\mathcal{H}_\theta$ be the completion of L_2 with respect to this (weaker) norm.

If $f \in L_2$ then $A^{is} f(x) = e^{isx} f(x)$ so that if $g = A^{is} f$ then $\hat{g}(\xi) = \hat{f}(\xi - s)$. Hence

$$\|A^{is} f\|_\theta \leq e^{\theta|s|} \|f\|_\theta. \tag{2.1}$$

We now wish to show that A induces a sectorial operator on $\mathcal{H}_\theta$. We do this by simply checking that the appropriate resolvent operators extend boundedly and satisfy the necessary bounds. To be precise if for some $0 < \phi < \pi$ we show that the operators $\lambda R(\lambda, A) = \lambda(\lambda - A)^{-1}$ extend to be bounded on $\mathcal{H}_\theta$ and if further

$$\sup_{|\arg\lambda| \geq \phi} \|\lambda R(\lambda, A)\|_{\mathcal{H}_\theta} < \infty$$

then the operator A defined with domain $(I + A)^{-1}(\mathcal{H}_\theta)$ and range $A(I + A)^{-1}(\mathcal{H}_\theta)$ is necessarily sectorial with $\omega(A) \leq \phi$. The facts that the domain and range are dense and A is one-one follow quickly once one notes

$$\lim_{t\to 0+} tA(I+tA)^{-1}f = \lim_{t\to\infty}(I+tA)^{-1}f = 0 \qquad f\in\mathcal{H}_\theta.$$

This follows easily from the bounds on the resolvent and the fact it is true on the dense subset L_2 of $\mathcal{H}_\theta$. This principle will be used several times for different completions of L_2.

The appropriate bounds on the resolvent follow from (2.1) by a method similar to that of the proof of the Dore-Venni Theorem [**3**]. The argument only requires that a Hilbert space has the (UMD)-property, but in the next Lemma we give a slightly more general result.

LEMMA 2.1. *There exists a constant C so that if $m\in L^1\cap L^\infty(\mathbb{R})$ satisfies*

$$\int_{-\infty}^{\infty}|\hat{m}(\xi)|e^{\theta|\xi|}d\xi<\infty$$

then for $f\in L_2(\mathbb{R})$

$$\|mf\|_\theta \le C\left(\|m\|_\infty + \int_{|\xi|\ge 1}|\hat{m}(\xi)|\,e^{\theta|\xi|}\,d\xi\right)\|f\|_\theta. \tag{2.2}$$

PROOF. Let us split $m=m_0+m_1$ where $\hat{m}_0=\hat{m}\chi_{[-1,1]}$. Note that

$$\|m_1\|_\infty \le C_0\int_{|\xi|\ge 1}|\hat{m}(\xi)|\,e^{\theta|\xi|}\,d\xi$$

where $C_0=C_0(\theta)$. Hence

$$\|m_0\|_\infty \le C_1\left(\|m\|_\infty + \int_{|\xi|\ge 1}|\hat{m}(\xi)|\,e^{\theta|\xi|}\,d\xi\right). \tag{2.3}$$

Now if $f\in L_2$

$$m_0 f = \frac{1}{2\pi}\int_{-\infty}^{\infty}\hat{m}_0(s)A^{is}f\,ds$$

as a Bochner integral in $L_2(\mathbb{R})$. Hence

$$A^{-it}(m_0 f) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\hat{m}_0(s)A^{i(s-t)}f\,ds.$$

Let $F,G\in L_2(\mathbb{R};L_2)$ be defined by $F(t)=A^{-it}(m_0 f)\chi_{[-1,1]}$ and $G(t)=A^{-it}f\chi_{[-2,2]}$. Then by the above $F=(2\pi)^{-1}\hat{m}_0 * G$ and so $\|F\|\le$

$\|m_0\|_\infty\|G\|$. Hence

$$\|m_0 f\|_\theta \le e^\theta \left(\int_{-1}^{1} \|A^{it}(m_0 f)\|_\theta^2 dt\right)^{\frac12}$$
$$\le e^\theta \|m_0\|_\infty \left(\int_{-2}^{2} \|A^{it} f\|_\theta^2 dt\right)^{\frac12}$$
$$\le 2e^{3\theta}\|m_0\|_\infty \|f\|_\theta,$$

where the last estimate follows from the fact that $\|A^{it}f\|_\theta \le e^{2\theta}\|f\|_\theta$ for $|t| \le 2$. In view of (2.3) we have

$$\|m_0 f\|_\theta \le C_2 \left(\|m\|_\infty + \int_{|\xi|\ge 1} |\hat{m}(\xi)|e^{\theta|\xi|}d\xi\right)\|f\|_\theta, \tag{2.4}$$

where $C_2 = C_2(\theta)$. On the other hand

$$m_1 f = \int_{|s|\ge 1} \hat{m}(s) A^{is} f ds$$

so that

$$\|m_1 f\|_\theta \le \left(\int_{|s|\ge 1} |\hat{m}(s)|e^{\theta|s|}ds\right)\|f\|_\theta.$$

Combining with (2.4) gives the Lemma. □

LEMMA 2.2. *A naturally extends to a sectorial operator on $\mathcal{H}_\theta$, which has an $H^\infty-$calculus with $\omega(A) = \omega_H(A) = \theta$.*

PROOF. Let us start from the formula

$$\int_{-\infty}^{\infty} \frac{e^{zx}}{1+e^x}dx = \frac{\pi}{\sin \pi z} \qquad 0 < \Re z < 1.$$

Hence if $t \in \mathbb{R}$

$$\int_{-\infty}^{\infty} \frac{e^{zx}}{e^t+e^x}dx = \frac{\pi e^{t(z-1)}}{\sin \pi z} \qquad 0 < \Re z < 1.$$

By analytic continuation we obtain that for any w in the complex plane with the negative real axis removed,

$$\int_{-\infty}^{\infty} \frac{e^{zx}}{w+e^x}dx = \frac{\pi w^{z-1}}{\sin \pi z} \qquad 0 < \Re z < 1.$$

Now let $m_{a,w}(x) = w^{1-a}e^{ax}(w+e^x)^{-1}$ where $0 < a < 1$. Then

$$\hat{m}_{a,w}(\xi) = \frac{\pi w^{-i\xi}}{\sin \pi(a-i\xi)}.$$

It follows from Lemma 2.1 that we have a uniform estimate

$$\|m_{a,w} f\|_\theta \le C\|f\|_\theta \qquad f \in L_2$$

as long as $|\arg w| + \theta < \pi - \delta$ for some $\delta > 0$. Here C depends on δ but not on a. We can let $a \to 0$ and deduce a similar estimate for $m_{0,w} = w(w + e^x)^{-1}$. Hence if we consider the resolvent operators

$$R(\lambda, A) = (\lambda - A)^{-1}$$

we obtain a uniform bound

$$\|\lambda R(\lambda, A)f\|_\theta \leq C\|f\|_\theta \qquad f \in L_2$$

as long as $|\arg \lambda| \geq \theta + \delta$ for some $\delta > 0$. This implies that we can naturally extend A to be sectorial on $\mathcal{H}_\theta$ and $\omega(A) \leq \theta$. Now, by the result of McIntosh [4] since $\mathcal{H}_\theta$ is a Hilbert space (2.1) implies that A admits an H^∞–calculus and $\omega_H(A) = \omega(A)$. □

We now introduce a new space by defining the norm

$$\|f\|_{X_\theta} := \sup_{a\in\mathbb{R}} \|f\chi_{(-\infty,a]}\|_\theta.$$

The space X_θ is defined as the completion of L_2 with respect to this norm. Note for $f \in L_2$ we have

$$\|f\|_\theta \leq \|f\|_{X_\theta} \leq \|f\|_0.$$

For $a \neq 0$ and $m, n \in \mathbb{N}$ we define the operator $E(m,n,a)$ on L_2 by

$$E(m,n,a)f(x) = \frac{1}{\sqrt{n}} \sum_{k=1}^{n} f(x - mka).$$

LEMMA 2.3. *For any $f \in L_2$ we have*

$$\lim_{n\to\infty} \limsup_{m\to\infty} \|E(m,n,a)f\|_{X_\theta} = \|f\|_\theta.$$

PROOF. Suppose $\alpha = 0$ or $\alpha = \theta$. First we note that

$$\|E(m,n,a)f\|_\alpha \leq \sqrt{n}\|f\|_\alpha \qquad f \in L_2(\mathbb{R}). \tag{2.5}$$

Now fix n, a and let $g_m = E(m,n,a)f$. Then

$$\hat{g}_m(\xi) = \frac{1}{\sqrt{n}} \hat{f}(\xi) \sum_{k=1}^{n} e^{-imka}.$$

Hence

$$\|g_m\|_\alpha^2 = \frac{1}{n} \int_{-\infty}^{\infty} (\sum_{j=1}^{n}\sum_{k=1}^{n} e^{i(j-k)ma})|\hat{f}(\xi)|^2 e^{-2\alpha|\xi|} d\xi.$$

By the Riemann-Lebesgue Lemma we obtain

$$\lim_{m\to\infty} \|E(m,n,a)f\|_\alpha = \|f\|_\alpha. \tag{2.6}$$

Now suppose $f \in L_2$ and $\epsilon > 0$. Fix M so large that

$$\|f - f\chi_{[-M,M]}\|_0 < \epsilon.$$

Let $f_0 = f\chi_{[-M,M]}$ and $f_1 = f - f_0$.

If $m > 2M|a|^{-1}$ then any $t \in \mathbb{R}$ falls in the support of at most one of the functions $f_0(x - mka)$ for $k = 1, 2, \dots$. Hence for any n we have for some $0 \le k \le n$,

$$\|\chi_{(-\infty,t)}E(m,n,a)f_0\|_\theta \le (k/n)^{\frac{1}{2}}\|E(m,k,a)f_0\|_\theta + n^{-\frac{1}{2}}\|f_0\|_0.$$

(If $k = 0$ we interpret $E(m, 0, a)f$ as 0). This shows that

$$\|E(m,n,a)f_0\|_{X_\theta} \le \max_{0\le k\le n}(k/n)^{\frac{1}{2}}\|E(m,k,a)f_0\|_\theta + n^{-\frac{1}{2}}\|f_0\|_0.$$

In view of (2.5) and (2.6) this gives

$$\limsup_{m\to\infty}\|E(m,n,a)f_0\|_{X_\theta} \le \|f_0\|_\theta + n^{-\frac{1}{2}}\|f_0\|_0. \tag{2.7}$$

On the other hand

$$\limsup_{m\to\infty}\|E(m,n,a)f_1\|_0 = \|f_1\|_0 < \epsilon$$

so that combining with (2.7) gives

$$\limsup_{m\to\infty}\|E(m,n,a)f\|_{X_\theta} \le \|f_0\|_\theta + n^{-\frac{1}{2}}\|f_0\|_0 + \epsilon. \tag{2.8}$$

Since $\|f_0\|_\theta \le \|f\|_\theta + \|f_1\|_\theta < \|f\|_\theta + \epsilon$ since obtain

$$\limsup_{n\to\infty}\limsup_{m\to\infty}\|E(m,n,a)f\|_{X_\theta} \le \|f\|_\theta + 2\epsilon.$$

Since the X_θ−norm is larger than the norm $\|\cdot\|_\theta$ this equation and (2.6) imply the conclusion. □

THEOREM 2.4. *The operator A on X_θ is sectorial and admits an H^∞−calculus but $\omega(A) = 0$ and $\omega_H(A) = \theta$.*

PROOF. For $\lambda \in \mathbb{C} \setminus \mathbb{R}_+$ let $m_\lambda(x) = \lambda(\lambda - e^x)^{-1}$. Then for $f \in L_2$

$$m_\lambda f = \int_{-\infty}^{\infty} \frac{\lambda e^x}{(\lambda - e^x)^2} f\chi_{(-\infty,x)}dx$$

as a Bochner integral in L_2. Hence if $\psi = \arg\lambda$,

$$\|m_\lambda f\|_{X_\theta} \le \|f\|_{X_\theta}\int_{-\infty}^{\infty}\frac{|\lambda|e^x}{|\lambda - e^x|^2}dx.$$

Now

$$\int_{-\infty}^{\infty}\frac{|\lambda|e^x}{|\lambda-e^x|^2}dx=\int_0^\infty\frac{|\lambda|}{|t-\lambda|^2}dt$$
$$=\int_0^\infty|t-e^{i\psi}|^{-2}dt.$$

Now reasoning as before we can deduce that $\lim_{t\to 0+}tA(I+tA)^{-1}f=\lim_{t\to\infty}(I+tA)^{-1}f=0$ for $f\in X_\theta$ by a density argument since it is true for $f\in L_2$. It follows that A is sectorial on X_θ and $\omega(A)=0$.

For any $m\in L^\infty(\mathbb{R})$ note that if $f\to mf$ extends to a bounded operator on $\mathcal{H}_\theta$ then that for $f\in L_2$ we have

$$\|mf\|_{X_\theta}=\sup_{-\infty<t<\infty}\|mf\chi_{(-\infty,t)}\|_\theta\le C\|f\|_{X_\theta}.$$

It follows that on X_θ, A has an $H^\infty-$calculus and $\omega_H(A)\le\theta$. It remains to show that $\omega_H(A)\ge\theta$. To do this, we show that for any s, $\|A^{is}\|_{X_\theta}=\|A^{is}\|_{\mathcal{H}_\theta}=e^{\theta|s|}$.

Suppose $s>0$ and let $a=2\pi/s$. For any $f\in L_2$ and $m,n\in\mathbb{N}$, we note that

$$\|A^{is}E(m,n,a)f\|_{X_\theta}\le\|A^{is}\|_{X_\theta}\|E(m,n,a)f\|_{X_\theta}.$$

Note by choice of a we have $A^{is}E(m,n,a)f=E(m,n,a)A^{is}f$ and so by Lemma 2.3

$$\|A^{is}f\|_\theta\le\|A^{is}\|_{X_\theta}\|f\|_\theta$$

and this shows $\|A^{is}\|_{X_\theta}=\|A^{is}\|_{\mathcal{H}_\theta}$ and completes the proof. □

We conclude by showing that we can use this example to produce a similar example modelled on a super-reflexive space. For this we will use complex interpolation. For $0<\tau<1$ we consider the complex interpolation space $X_{\theta,\tau}=[L_2,X_\theta]_\tau$. Let us recall the definition of this space. Let $\mathcal{S}$ denote the strip $0<\Re z<1$. We consider the vector space $\mathcal{F}$ of all bounded continuous functions $F:\overline{\mathcal{S}}\to X_\theta$ which are analytic on $\mathcal{S}$ and such that $F(it)\in L_2$ for $-\infty<t<\infty$ and $t\to F(it)$ is continuous into L_2. We norm $\mathcal{F}$ by

$$\|F\|_{\mathcal{F}}=\max(\sup_{-\infty<t<\infty}\|F(it)\|_0,\ \sup_{-\infty<t<\infty}\|F(1+it)\|_{X_\theta}).$$

We then define $X_{\theta,\tau}$ to be the space of all $f\in X_\theta$ such that for some $F\in\mathcal{F}$ we have $F(\tau)=f$ under the norm

$$\|f\|_{X_{\theta,\tau}}=\inf\{\|F\|_{\mathcal{F}}:\ F(\tau)=f\}.$$

We will need the following fact about complex interpolation. Let $P:\partial\mathcal{S}\times\mathcal{S}\to\mathbb{R}$ be the Poisson kernel for the strip. Given τ let $h_0(t)=P(it,\tau)$ and $h_1(t)=P(1+it,\tau)$. Thus the measure on $\partial\mathcal{S}$ given by

$h_0(t)dt$ on the line $i\mathbb{R}$ and $h_1(t)dt$ on the line $1+i\mathbb{R}$ is harmonic measure for the point τ. Then h_0, h_1 are non-negative continuous functions in $L_1(\mathbb{R})$ with

$$\int_{-\infty}^{\infty} (h_0(t) + h_1(t))\, dt = 1$$

such that if $F \in \mathcal{F}$ then

$$\|F(\tau)\|_{X_{\theta,\tau}} \leq \int_{-\infty}^{\infty} (\|F(it)\|_0 h_0(t) + \|F(1+it)\|_{X_\theta} h_1(t))\, dt. \tag{2.9}$$

This estimate goes back to Calderón [**1**].

It follows immediately by interpolation that A induces a sectorial operator on $X_{\theta,\tau}$ with $\omega(A) = 0$. Indeed (1.1) for any $\phi > 0$ is immediate and we can deduce that

$$\lim_{t\to 0+} tA(I+tA)^{-1}f = \lim_{t\to\infty} (I+tA)^{-1}f = 0$$

for every $f \in X_{\theta,\tau}$ either by a standard density argument or by the remarks above. Indeed if F is admissible then, for example, we have

$$\lim_{t\to 0+} \|tA(I+tA)^{-1}F(is)\|_0 = \lim_{t\to 0+} \|tA(I+tA)^{-1}F(1+is)\|_{X_\theta} = 0$$

if $-\infty < s < \infty$ and so by (2.9) and the Dominated Convergence Theorem

$$\lim_{t\to 0} \|tA(1+tA)^{-1}F(\tau)\|_{X_{\theta,\tau}} = 0.$$

Interpolation also quickly yields that the sectorial operator A on $X_{\theta,\tau}$ has an H^∞–calculus with $\omega_H(A) \leq \theta$. The spaces $X_{\theta,\tau}$ for $0 < \tau < 1$ are uniformly convex (and thus super-reflexive). We now show that on these spaces we also have $\omega_H(A) > \omega(A)$.

PROPOSITION 2.5. *On $X_{\theta,\tau}$ we have $\omega_H(A) = \tau\theta$.*

PROOF. By interpolation we have $\|A^{is}\|_{X_{\theta,\tau}} \leq e^{\tau\theta|s|}$. We shall show that $\|A^{is}\|_{X_{\theta,\tau}} = e^{\tau\theta|s|}$ and by Theorem 5.4 of [**2**] this will imply that $\omega_H(A) = \tau\theta$.

We need the fact that if $f \in L_2$ then $\|f\|_{X_{\theta,\tau}} \geq \|f\|_{\tau\theta}$. This follows immediately from the fact that $\|f\|_{X_\theta} \geq \|f\|_\theta$ and $[L_2, \mathcal{H}_\theta]_\tau = \mathcal{H}_{\tau\theta}$.

Fix $s \neq 0$ and let $a = 2\pi/s$.

Suppose $f \in L_2$ is such that

$$\int |\hat{f}(\xi)|^2 e^{2\theta|\xi|} d\xi < \infty.$$

Define $F : \mathcal{S} \to L_2$ be defined by

$$\widehat{F(z)}(\xi) = e^{\theta(z-\tau)}\hat{f}(\xi).$$

Thus F extends continuously to ∂S and $\|F(it)\|_0 = \|F(1+it)\|_\theta = \|f\|_{\tau\theta}$. Then using (2.9)

$$\|E(m,n,a)f\|_{X_{\theta,\tau}} \leq$$
$$\int_{-\infty}^{\infty} \|E(m,n,a)F(it)\|_0 h_0(t) + \|E(m,n,a)F(1+it)\|_{X_\theta} h_1(t)\, dt.$$

If we fix n and let $m \to \infty$ we can use the Dominated Convergence Theorem and (2.7) to deduce that

$$\limsup_{m\to\infty} \|E(m,n,a)f\|_{X_{\theta,\tau}} \leq$$
$$\int_{-\infty}^{\infty} (\|F(it)\|_0 h_0(t) + \|F(1+it)\|_\theta h_1(t) + n^{-\frac{1}{2}} \|F(1+it)\|_0 h_1(t))dt.$$

By (2.6) we have

$$\lim_{m\to\infty} \|E(m,n,a)f\|_{\tau\theta} = \|f\|_{\tau\theta}.$$

Hence, letting $n \to \infty$ we obtain

$$\lim_{n\to\infty} \limsup_{m\to\infty} \|E(m,n,a)f\|_{X_{\theta,\tau}} = \|f\|_{\tau\theta}.$$

Thus the analogue of Lemma 2.3 holds at least for f in a dense subset of L_2 (which is itself dense in $X_{\theta,\tau}$.) Hence arguing as before in Theorem 2.4 we obtain that

$$\|A^{is}\|_{X_{\theta,\tau}} \geq \|A^{is}\|_{\mathcal{H}_{\tau\theta}} = e^{\tau\theta|s|}.$$

This completes the proof. □

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